YEAR 10 – SIMILARITY...

Congruence, similarity & enlargement

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Enlarge by a positive scale factor
- Enlarge by a fractional scale factor
- Identify similar shapes
- Work out missing sides and angles in similar shapes
- Use parallel lines to find missing angles
- Understand similarity and congruence

Keywords

Enlarge: to make a shape bigger (or smaller) by a given multiplier (scale factor)
Scale Factor: the multiplier of enlargement
Centre of enlargement: the point the shape is enlarged from
Similar: when one shape can become another with a reflection, rotation, enlargement or translation.
Congruent: the same size and shape
Corresponding: items that appear in the same place in two similar situations
Parallel: straight lines that never meet (equal gradients)

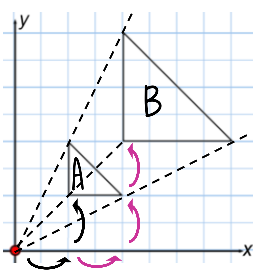
Positive scale factors (R)

Enlargement from a point

Enlarge shape A by SF 2 from (0,0)

The shape is enlarged by 2

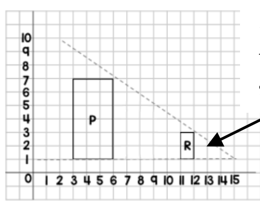
The distance from the point enlarges by 2



Fractional scale factors (R)

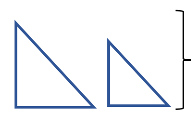
Fractions less than 1 make a shape SMALLER

R is an enlargement of P by a scale factor $\frac{1}{3}$ from centre of enlargement (15,1)



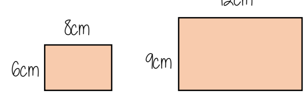
SF: $\frac{1}{3}$ - R is three times smaller than P

Identify similar shapes



Angles in similar shapes do not change.
e.g if a triangle gets bigger the angles can not go above 180°

Similar shapes

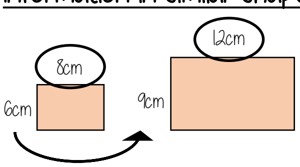


Scale Factor: Both sides on the bigger shape are 1.5 times bigger

Compare sides: 6 : 9 → 2 : 3   8 : 12 → 2 : 3

Both sets of sides are in the same ratio

Information in similar shapes



Compare the equivalent side on both shapes

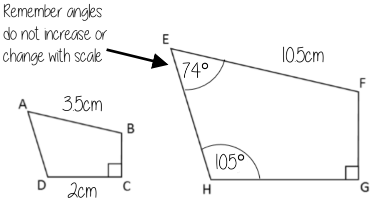
Scale Factor is the multiplicative relationship between the two lengths

Shape ABCD and EFGH are similar

Notation helps us find the corresponding sides

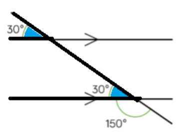
AB and EF are corresponding

Remember angles do not increase or change with scale



Angles in parallel lines (R)

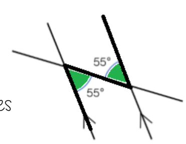
Alternate angles



Because alternate angles are equal the highlighted angles are the same size

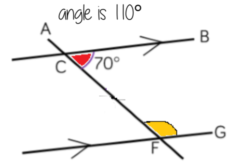
Corresponding angles

Because corresponding angles are equal the highlighted angles are the same size



Co-interior angles

Because co-interior angles have a sum of 180° the highlighted angle is 110°

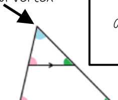


As angles on a line add up to 180° co-interior angles can also be calculated from applying alternate/ corresponding rules first

Similar triangles

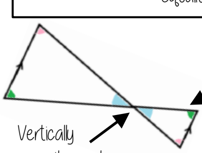
Shares a vertex

Because corresponding angles are equal the highlighted angles are the same size



Parallel lines – all angles will be the same in both triangle

As all angles are the same this is similar – it only one pair of sides are needed to show equality

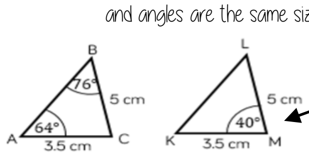


All the angles in both triangles are the same and so similar

Vertically opposite angles

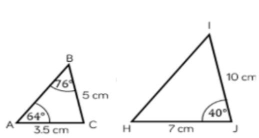
Congruence and Similarity

Congruent shapes are identical – all corresponding sides and angles are the same size



$A\hat{C}B = K\hat{M}L$

Because all the angles are the same and AC=KM BC=LM triangles ABC and KLM are **congruent**



Because all angles are the same, but all sides are enlarged by 2 ABC and HIJ are **similar**

Conditions for congruent triangles

Triangles are congruent if they satisfy any of the following conditions

Side-side-side

All three sides on the triangle are the same size

Angle-side-angle

Two angles and the side connecting them are equal in two triangles

Side-angle-side

Two sides and the angle in-between them are equal in two triangles (it will also mean the third side is the same size on both shapes)

Right angle-hypotenuse-side

The triangles both have a right angle, the hypotenuse and one side are the same

YEAR 10 – SIMILARITY...

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Trigonometry

What do I need to be able to do?

By the end of this unit you should be able to:

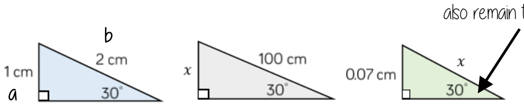
- Work fluently with hypotenuse, opposite and adjacent sides
- Use the tan, sine and cosine ratio to find missing side lengths
- Use the tan, sine and cosine ratio to find missing angles
- Calculate sides using Pythagoras' Theorem

Keywords

Enlarge: to make a shape bigger (or smaller) by a given multiplier (scale factor)
Scale Factor: the multiplier of enlargement
Constant: a value that remains the same
Cosine ratio: the ratio of the length of the adjacent side to that of the hypotenuse. The sine of the complement.
Sine ratio: the ratio of the length of the opposite side to that of the hypotenuse.
Tangent ratio: the ratio of the length of the opposite side to that of the adjacent side.
Inverse: function that has the opposite effect.
Hypotenuse: longest side of a right-angled triangle. It is the side opposite the right-angle.

Ratio in right-angled triangles

When the angle is the same the ratio of sides a and b will also remain the same



a : b
1 : 2

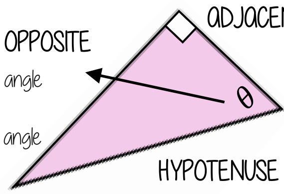
$\frac{a}{b} = \frac{1}{2}$

a : b
x : 100
50 : 100

a : b
0.07 : x
0.07 : 0.14

Hypotenuse, adjacent and opposite

ONLY right-angled triangles are labelled in this way



OPPOSITE: Always opposite an acute angle. Useful to label second. Position depend upon the angle in use for the question

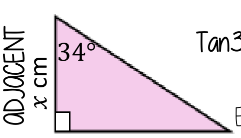
ADJACENT: Next to the angle in question. Often labelled last

HYPOTENUSE: Always the longest side. Always opposite the right angle. Useful to label this first

Tangent ratio: side lengths

$\text{Tan}\theta = \frac{\text{opposite side}}{\text{adjacent side}}$

Substitute the values into the tangent formula



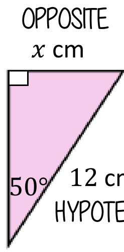
$\text{Tan}34 = \frac{10}{x}$

Equations might need rearranging to solve

$x \times \text{Tan}34 = 10$

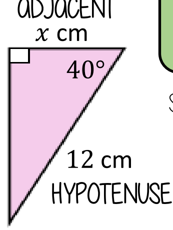
$x = \frac{10}{\text{Tan}34} = 14.8\text{cm}$

Sin and Cos ratio: side lengths



$\text{Sin}\theta = \frac{\text{opposite side}}{\text{hypotenuse side}}$

NOTE
The Sin(x) ratio is the same as the Cos(90-x) ratio



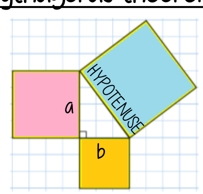
$\text{Cos}\theta = \frac{\text{adjacent side}}{\text{hypotenuse side}}$

Substitute the values into the ratio formula

Equations might need rearranging to solve

Pythagoras theorem

R



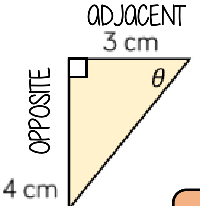
$\text{Hypotenuse}^2 = a^2 + b^2$

This is commutative – the square of the hypotenuse is equal to the sum of the squares of the two shorter sides

Places to look out for Pythagoras
- Perpendicular heights in isosceles triangles
- Diagonals on right angled shapes
- Distance between coordinates
- Any length made from a right angles

Sin, Cos, Tan: Angles

Inverse trigonometric functions



Label your triangle and choose your trigonometric ratio

Substitute values into the ratio formula

$\theta = \text{Tan}^{-1}\frac{\text{opposite side}}{\text{adjacent side}}$

$\theta = \text{Sin}^{-1}\frac{\text{opposite side}}{\text{hypotenuse side}}$

$\theta = \text{Cos}^{-1}\frac{\text{adjacent side}}{\text{hypotenuse side}}$

$\text{Tan}\theta = \frac{3}{4}$

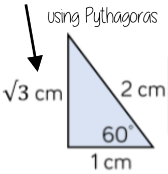
$\theta = \text{Tan}^{-1}\frac{3}{4}$

$\theta = 36.9°$

Key angles

Because trig ratios remain the same for similar shapes you can generalise from the following statements

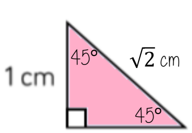
This side could be calculated using Pythagoras



$\text{Tan}30 = \frac{1}{\sqrt{3}}$
$\text{Tan}60 = \sqrt{3}$

$\text{Cos}30 = \frac{\sqrt{3}}{2}$
$\text{Cos}60 = \frac{1}{2}$

$\text{Sin}30 = \frac{1}{2}$
$\text{Sin}60 = \frac{\sqrt{3}}{2}$



$\text{Tan}45 = 1$

$\text{Cos}45 = \frac{1}{\sqrt{2}}$

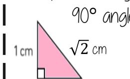
$\text{Sin}45 = \frac{1}{\sqrt{2}}$

Key angles 0° and 90°

$\text{Tan}0 = 0$

~~Tan90~~

This value cannot be defined – it is impossible as you cannot have two 90° angles in a triangle



$\text{Sin}0 = 0$
$\text{Sin}90 = 1$
$\text{Cos}0 = 1$
$\text{Cos}90 = 0$

YEAR 10 – DEVELOPING ALGEBRA...

Representing solutions of equations and inequalities

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What do I need to be able to do?

By the end of this unit you should be able to:

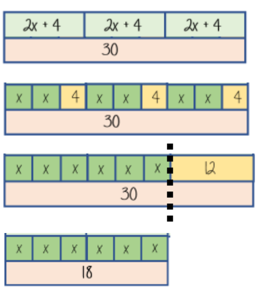
- Form and solve equations and inequalities
- Represent and interpret solutions on a number line as inequalities
- Draw straight line graphs and find solutions to equations
- Form and solve equations and inequalities with unknowns on both sides

Keywords

Solution: a value we can put in place of a variable that makes the equation true
Variable: a symbol for a number we don't know yet
Equation: an equation says that two things are equal – it will have an equals sign =
Expression: numbers, symbols and operators grouped together to show the value of something
Identity: On equation where both sides have variables that cause the same answer includes ≡
Linear: an equation or function that is the equation of a straight line
Intersection: the point that two lines meet
Inequality: an inequality compares two values showing if one is greater than, less than or equal to another.

Solve equations (R)

$3(2x + 4) = 30$



$3(2x + 4) = 30$

Expand the brackets

$6x + 12 = 30$

-12 -12

Substitute to check your answer. This could be negative or a fraction or decimal

$6x = 18$

$\div 6$ $\div 6$

$x = 3$

Form and solve inequalities (R)

Two more than treble my number is greater than 11

Form

x → ×3 → +2 → 11

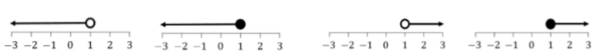
$3x + 2 > 11$

Solve

x ← ÷3 ← −2 ← 11

$x > 3$

Solutions on a number line

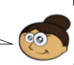


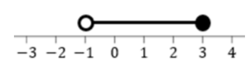
$x < 1$ $x \le 1$ — Both represent values less than 1. Includes the value 1

$x > 1$ $x \ge 1$ — Both represent values more than 1. Includes the value 1

● Includes the value it sits above
○ Does NOT include the value it sits above

Values less than or equal to 3 but also more than -1





$-1 < x \le 3$

This includes the integer values 0,1,2,3

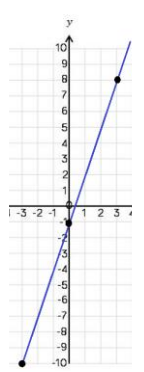
Plotting straight line graphs (R)

$y = 3x - 1$ → 3 x the x coordinate then – 1

x	−3	0	3
y	-10	-1	8

Draw a table to display this information

This represents a coordinate pair (-3, -10)



You only need two points to form a straight line

Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

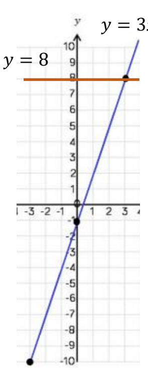
Remember to join the points to make a line

Find solutions graphically

For linear equations there is only one point the graph meets the x value

$x = 2$
$y = 4$

These two lines will cross at (2,4) because they are just x= and y= they are parallel to axes and meet in one place



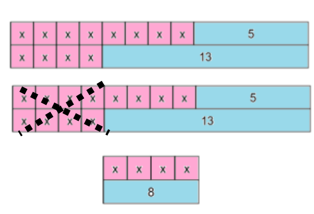
$3x - 1 = 8$

Remember equation of a line format is y = mx + c

The solution is the point the two lines meet (3,8)

Equations: unknown on both sides (R)

$8x + 5 = 4x + 13$



$8x + 5 = 4x + 13$
$-4x$ $-4x$
$4x + 5 = 13$
-5 -5
$4x = 8$
$\div 4$ $\div 4$
$x = 2$

Inequalities: unknown on both sides

$8x + 5 \le 4x + 13$ → $x \le 2$

Any value 2 or less will satisfy this inequality

YEAR 10 – DEVELOPING ALGEBRA…

Simultaneous Equations

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What do I need to be able to do?

By the end of this unit you should be able to:

- Determine whether (x,y) is a solution
- Solve by substituting a known variable
- Solve by substituting an expression
- Solve graphically
- Solve by subtracting/ adding equations
- Solve by adjusting equations
- Form and solve linear simultaneous equations

Keywords

Solution: a value we can put in place of a variable that makes the equation true
Variable: a symbol for a number we don't know yet.
Equation: an equation says that two things are equal – it will have an equals sign =
Substitute: replace a variable with a numerical value
LCM: lowest common multiple (the first time the times table of two or more numbers match)
Eliminate: to remove
Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)
Coordinate: a set of values that show an exact position.
Intersection: the point two lines cross or meet.

Is (x, y) a solution?

x and y represent values that can be substituted into an equation

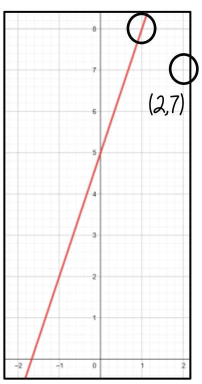
Does the coordinate (1,8) lie on the line y=3x+5?

This coordinate represents x=1 and y=8

$y = 3x + 5$

$8 = 3(1) + 5$

As the substitution makes the equation correct the coordinate (1,8) IS on the line y=3x+5



Is (2,7) on the same line?

$7 \neq 3(2) + 5$

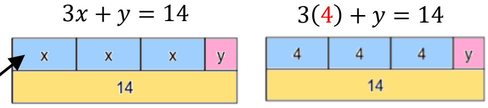
No 7 does NOT equal 6+5

Substituting known variables

A line has the equation 3x + y = 14

Stephanie knows the point x = 4 lies on that line. Find the value for y

Two different variables, two solutions



$x = 4$

$3x + y = 14$

$3(4) + y = 14$

$12 + y = 14$

$-12 \quad -12$

$y = 2$

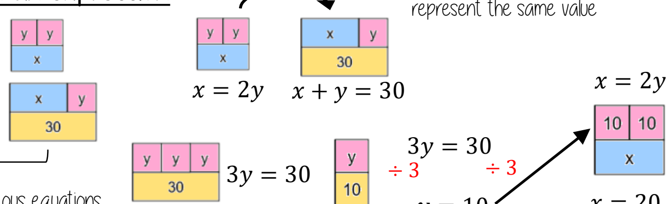
Substituting in an expression

x = 2y

x + y = 30

Pair of simultaneous equations (two representations)

Substitute 2y in place of the x variable as they represent the same value



$x = 2y \qquad x + y = 30$

$3y = 30$

$3y = 30$

$\div 3 \quad \div 3$

$y = 10$

$x = 2y$

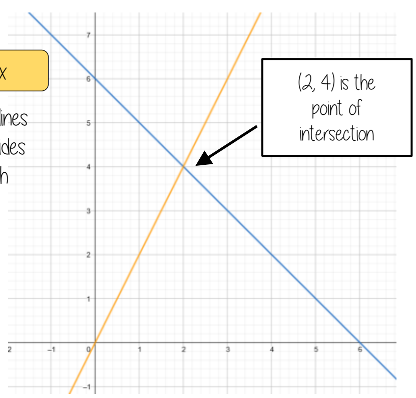
$x = 20$

Solve graphically

x + y = 6

y = 2x

Linear equations are straight lines. The point of intersection provides the x and y solution for both equations

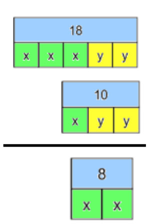


(2, 4) is the point of intersection

The solution that satisfies both equations is

$x = 2$ and $y = 4$

Solve by subtraction



$3x + 2y = 18$

$-\quad x + 2y = 10$

$2x = 8$

$\div 2 \quad \div 2$

$x = 4$

$x + 2y = 10$

$(4) + 2y = 10$

$-4 \quad -4$

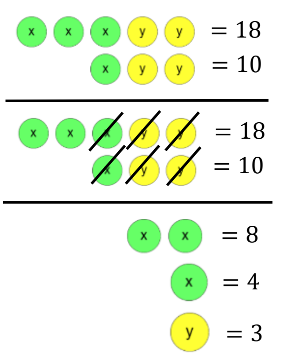
$2y = 6$

$\div 2 \quad \div 2$

$y = 3$

$x = 4$

$y = 3$



Solve by addition

$3x + 2y = 16$

$+\ 6x - 2y = 2$

$9x = 18$

$\div 9 \quad \div 9$

$x = 2$

$3x + 2y = 16$

$3(2) + 2(y) = 16$

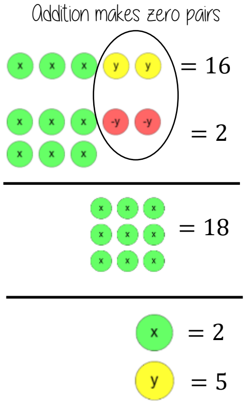
$6 + 2y = 16$

$-6 \quad -6$

$2y = 10$

$y = 5$

Addition makes zero pairs



Solve by adjusting one

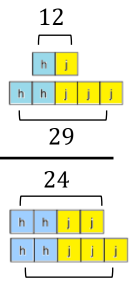
$h + j = 12$

$2h + 2j = 29$

No equivalent values

$2h + 2j = 24$

$2h + 2j = 29$



By proportionally adjusting one of the equations – now solve the simultaneous equations choosing an addition or subtraction method

Solve by adjusting both

$2x + 3y = 39$

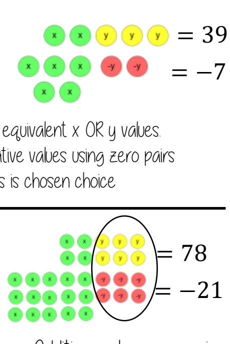
$5x - 2y = -7$

Use LCM to make equivalent x OR y values. Because of the negative values using zero pairs and y values is chosen choice

$4x + 6y = 78$

$15x - 6y = -21$

Now solve by addition



Addition makes zero pairs

YEAR 10 – GEOMETRY...

@whisto_maths

Angles and bearings

What do I need to be able to do?

By the end of this unit you should be able to:

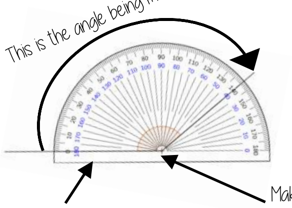
- Understand and represent bearings
- Measure and read bearings
- Make scale drawings using bearings
- Calculate bearings using angle rules
- Solve bearings problems using Pythagoras and trigonometry

Keywords

Cardinal directions: the directions of North, South, East, West
Angle: the amount of turn between two lines around their common point
Bearing: the angle in degrees measured clockwise from North.
Perpendicular: where two lines meet at 90°
Parallel: straight lines always the same distance apart and never touch. They have the same gradient.
Clockwise: moving in the direction of the hands on a clock.
Construct: to draw accurately using a compass, protractor and or ruler or straight edge.
Scale: the ratio of the length of a drawing to the length of the real thing
Protractor: an instrument used in measuring or drawing angles.

Measure angles to 180° (R)

This is the angle being measured



Read from 0° on the base line. Remember to use estimation. This is an obtuse angle so between 90 ° and 180 °

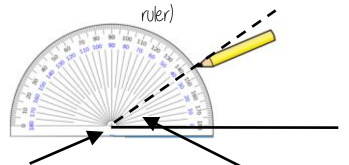
The base line follows the line segment

Make sure the cross is at the point the two lines meet

Draw angles up to 180° (R)

Draw a 35° angle

Make a mark at 35° with a pencil. And join to the angle point (use a ruler)

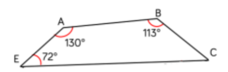


Make sure the cross is at the end of the line (where you want the angle)

The angle

Angle notation (R)

The letter in the middle is the angle
The arc represents the part of the angle



Angle Notation: three letters $A\hat{B}C$ This is the angle at B = 113 °
$\angle ABC$ is also used to represent the angle at B.

Scale drawings (R)

1 : 20

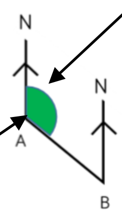
For every 1cm on the model there are 20cm in real life

Remember: Scale drawings ONLY change lengths and distances. Angles remain the same

Understand and represent bearings

- A bearing is always measured from NORTH
- It is always given as three figures

The angle indicated starts from the North line at A and joins the path connecting A to B.



This angle shows the bearing of **B from A**

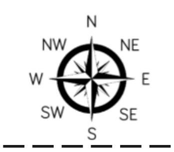
The bearing of B from A is calculated by measuring the highlighted angle

The sentence… "Bearing of ____ from _____" is really important in identifying the bearing being represented

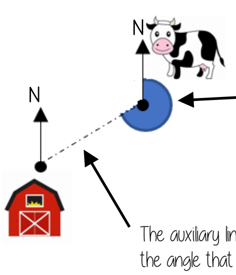
Using <u>estimation</u> it is clear this angle is between 090° and 180°

Directions

Clockwise   Anti-Clockwise



Measure and read bearings



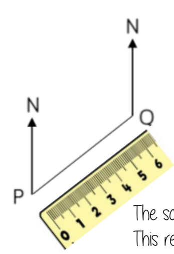
The bearing of the cow to the barn

This angle is measured from NORTH
It is measured in a clockwise direction
<u>Estimation</u> indicates this angle is between 180° and 270°
Use a protractor to measure accurately
Remember: bearings are written as three figures.

The auxiliary line is drawn to help you measure and draw the angle that is measured to represent the bearing

Scale drawings using bearings

Remember – angles DO NOT change size in scaled drawings



The bearing measurements do not change from "real life" to images

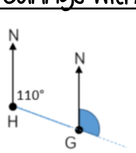
The units in the ratio scale are the same

The scale may need to be calculated from the image.
This represents 30km from P to Q

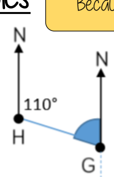
6cm = 30km
6:3,000,000

Bearings with angle rules

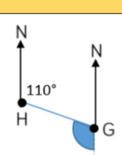
Because two North lines are PARALLEL….



They form <u>corresponding</u> <u>angles</u> and therefore are the same size



They form <u>co-interior</u> <u>angles</u> and add up to 180°

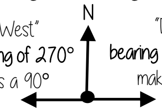


They form <u>alternate</u> <u>angles</u> and therefore are the same size

Bearings with right-angled geometry

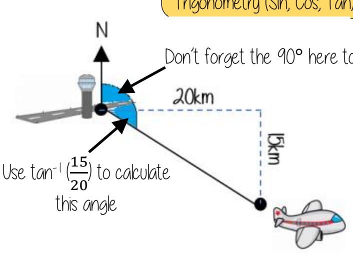
Look for Right-angles:
Pythagoras
Trigonometry (Sin, Cos, Tan)

"Due West" **bearing of 270°** makes a 90° angle



"Due East" **bearing of 090°** makes a 90° angle

A plane flies East for 20km then turns South for 15km. Find the bearing of the plane from where it took off.



Don't forget the 90° here too

Use $\tan^{-1}\left(\frac{15}{20}\right)$ to calculate this angle

YEAR 10 – GEOMETRY...

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Working with circles

What do I need to be able to do?

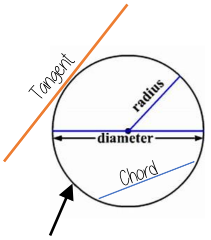
By the end of this unit you should be able to:

- Recognise and label parts of a circle
- Calculate fractional parts of a circle
- Calculate the length of an arc
- Calculate the area of a sector
- Understand and use volume of a cone, cylinder and sphere.
- Understand and use surface area of a cone, cylinder and sphere.

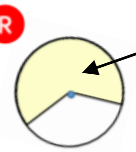
Keywords

Circumference: the length around the outside of the circle – the perimeter
Area: the size of the 2D surface
Diameter: the distance from one side of a circle to another through the centre
Radius: the distance from the centre to the circumference of the circle
Tangent: a straight line that touches the circumference of a circle
Chord: a line segment connecting two points on the curve
Frustum: a pyramid or cone with the top cut off
Hemisphere: half a sphere
Surface area: the total area of the surface of a 3D shape.

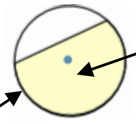
Parts of a circle



Circumference



Sector (part of the circle made from two radii)



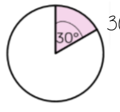
Segment (part of the circle made from a chord)

An arc is a part of the circumference

Fractional parts of a circle

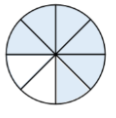
A circle is made up of 360°

Formula to remember:
Area of a circle = πr^2
Circumference of a circle = πd or $2\pi r$



30° represents $\frac{30}{360}$ of a full circle

$\frac{30}{360} = \frac{1}{12}$



$\frac{270}{360}$ of a full circle (in degrees)

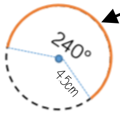
$\frac{6}{8}$ of a full circle (in equal parts)

$\frac{3}{4}$ of a full circle

The fraction of the circle is as $\frac{\theta}{360}$

θ represents the degrees in the sector

Arc length



Remember an arc is part of the circumference
Circumference of the whole circle = πd = $\pi \times 9 = 9\pi$

Arc length = $\frac{\theta}{360}$ ×circumference

$= \frac{240}{360} \times 9\pi$

$= \frac{2}{3} \times 9\pi$ $= \underline{6\pi}$

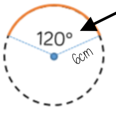
Perimeter

Perimeter is the length around the outside of the shape
This includes the arc length and the radii that encloses the shape

Perimeter = $\frac{\theta}{360}$ ×circumference $+ 2r$

$= \underline{6\pi + 9}$

Sector area



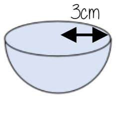
Remember a sector is part of a circle
Area of the whole circle = πr^2 = $\pi \times 6^2 = 36\pi$

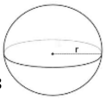
Sector area = $\frac{\theta}{360}$ × area of circle

$= \frac{120}{360} \times 36\pi$

$= \frac{1}{3} \times 36\pi$ $= \underline{12\pi}$

Volume of a sphere





Volume Sphere = $\frac{4}{3}\pi r^3$

NOTE: This is now a **cubed** value

Volume Sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 3^3$

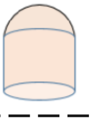
$= \frac{4}{3} \times \pi \times 27$ $= \underline{36\pi}$

A hemisphere is half the volume of the overall sphere

$= 36\pi \div 2$

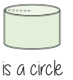
$= 18\pi$

Look out for hemispheres being placed on other 3D shapes, e.g cones and cylinders

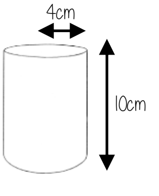


Volume of a cone and a cylinder

Volume Cylinder= $\pi r^2 h$



A cylinder is a prism – cross section is a circle



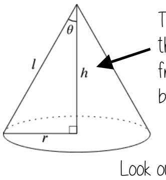
$V = \pi r^2 h$
$= \pi \times 4^2 \times 10$
$= \pi \times 160$
$= \underline{160\pi}$ cm²

Give your answer in terms of π' means NOT in terms of pi

$= \underline{502.7}$cm²

Volume Cone = $\frac{1}{3}\pi r^2 h$

A cone is a pyramid with a circular base

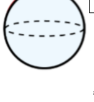


The height of a cone is the perpendicular height from the vertex to the base

Look out for trigonometry or Pythagoras theorem – the radius forms the base of a right-angled triangle

Surface area of a sphere

Surface area = $4\pi r^2$



Radius = 5cm

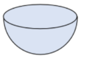
Surface area = $4\pi r^2$

$= 4 \times \pi \times 5^2$

$= 4 \times \pi \times 25$

The curved surface area of a sphere $= \underline{100\pi}$

A hemisphere has the curved surface AND a flat circular face



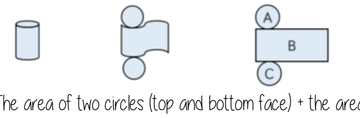
$= 100\pi \div 2 = 50\pi$

$= 50\pi + \pi \times 5^2$

Hemisphere $= \underline{75\pi}$

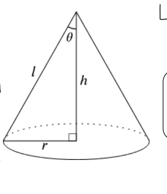
Surface area of cones and cylinders

Surface area cylinder= $2\pi r^2 + \pi dh$



The area of two circles (top and bottom face) + the area of the curved face

The length of shape B is the circumference of the circles



Curved surface area Cone = πrl

Look out for the use of Pythagoras to calculate the length l

Total surface area = curved face + circle face (area of base)

YEAR 10 – GEOMETRY…

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Vectors

What do I need to be able to do?

By the end of this unit you should be able to:

- Understand and represent vectors
- Use and read vector notation
- Draw and understand vectors multiplied by a scalar
- Draw and understand addition of vectors
- Draw and understand addition and subtraction of vectors

Keywords

Direction: the line our course something is going
Magnitude: the magnitude of a vector is its length
Scalar: a single number used to represent the multiplier when working with vectors
Column vector: a matrix of one column describing the movement from a point
Resultant: the vector that is the sum of two or more other vectors
Parallel: straight lines that never meet

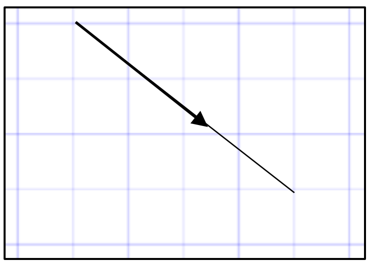
Understand and represent vectors

Column vectors have been seen in translations to describe the movement of one image onto another

Movement along the x-axis →

Movement along the y-axis →

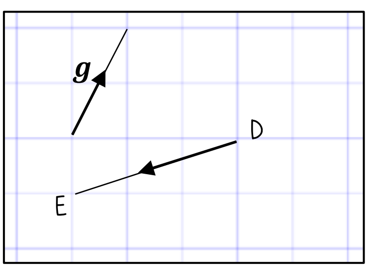
$\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ →



Vectors show both direction and magnitude

The arrow is pointing in the direction from starting point to end point of the vector.	The direction is important to correctly write the vector
The magnitude is the length of the vector (This is calculated using Pythagoras theorem and forming a right-angled triangle with auxiliary lines)	The magnitude stays the same even if the direction changes

Understand and represent vectors



Vector notation $\overrightarrow{DE}$ is another way to represent the vector joining the point D to the point E.

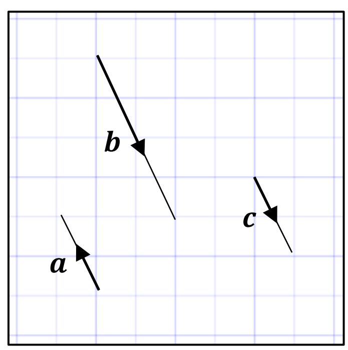
$\overrightarrow{DE} = \begin{pmatrix} -3 \\ -1 \end{pmatrix}$

The arrow also indicates the direction from point D to point E

Vectors can also be written in bold lower case so $\boldsymbol{g}$ represents the vector $\boldsymbol{g} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Vectors multiplied by a scalar

Parallel vectors are scalar multiples of each other



$\boldsymbol{a} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ $\boldsymbol{b} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$ $\boldsymbol{c} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$\boldsymbol{b} = 2 \times \boldsymbol{c} = 2\boldsymbol{c}$

Multiply $\boldsymbol{c}$ by 2 this becomes $\boldsymbol{b}$. The two lines are parallel

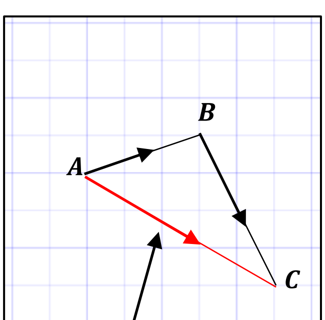
$\boldsymbol{a} = -1 \times \boldsymbol{c} = -\boldsymbol{c}$

The vectors $\boldsymbol{a}$ and $\boldsymbol{c}$ are also parallel. A negative scalar causes the vector to reverse direction.

$\boldsymbol{b} = -2 \times \boldsymbol{a} = -2\boldsymbol{a}$

Addition of vectors

$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ $\overrightarrow{BC} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$



The resultant

$\overrightarrow{AB} + \overrightarrow{BC}$

$= \begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix}$

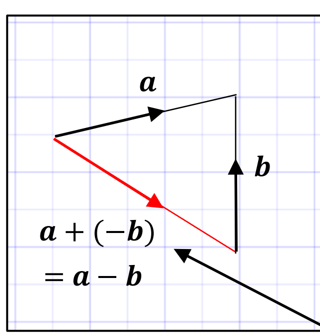
$= \begin{pmatrix} 3+2 \\ 1+-4 \end{pmatrix}$

$\overrightarrow{AC} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$

Look how this addition compares to the vector $\overrightarrow{AC}$

$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$

Addition and subtraction of vectors



$\boldsymbol{a} + (-\boldsymbol{b})$

$= \boldsymbol{a} - \boldsymbol{b}$

$\boldsymbol{a} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ $\boldsymbol{b} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$

$\boldsymbol{a} + (-\boldsymbol{b}) = \begin{pmatrix} 5+ & -0 \\ 1+ & -4 \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$

The resultant is $\boldsymbol{a} - \boldsymbol{b}$ because the vector is in the opposite direction to b which needs a scalar of -1

YEAR 10 – PROPORTION...

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Ratios and fractions

What do I need to be able to do?

By the end of this unit you should be able to:

- Compare quantities using ratio
- Link ratios and fractions and make comparisons
- Share in a given ratio
- Link Ratio and scales and graphs
- Solve problems with currency conversions
- Solve 'best buy' problems
- Combine ratios

Keywords

Ratio: a statement of how two numbers compare
Equivalent: of equal value
Proportion: a statement that links two ratios
Integer: whole number, can be positive, negative or zero.
Fraction: represents how many parts of a whole.
Denominator: the number below the line on a fraction. The number represent the total number of parts..
Numerator: the number above the line on a fraction. The top number. Represents how many parts are taken
Origin: (0,0) on a graph. The point the two axes cross
Gradient: The steepness of a line

Compare with ratio (R)

"For every dog there are 2 cats"

Dogs: Cats
1:2

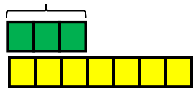
The ratio has to be written in the same order as the information is given.
e.g. 2:1 would represent 2 dogs for every 1 cat.

Units have the be of the same value to compare ratios

Ratios and fraction (R)

Trees: Flowers
3 : 7

Trees



Flowers

Ratio

Fraction of trees

$\frac{\text{Number of parts of in group}}{\text{Total number of parts}}$ → $\frac{3}{10}$

Fraction

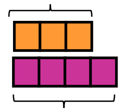
Sharing a whole into a given ratio (R)

James and Lucy share £350 in the ratio 3:4.
Work out how much each person earns

Model the Question

James: Lucy
3 : 4

James



Lucy
£350

Find the value of one part

Whole: £350
7 parts to share between
(3 James, 4 Lucy)

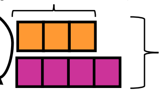
£350 ÷ 7 = £50

□ = one part = £50

Put back into the question

James: Lucy
x 50 3 : 4 x 50
£150:£200

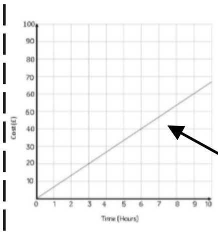
James = 3 x £50 = £150



£350

Lucy = 4 x £50 = £200

Ratio and graphs (R)



Graphs with a constant ratio are directly proportional

- Form a straight line
- Pass through (0,0)

The gradient is the constant ratio

Ratio and scale (R)

A picture of a car is drawn with a scale of 1:30

The car image is 10cm

Image : Real life
x 10 1cm : 30cm x 10
10cm : 300cm

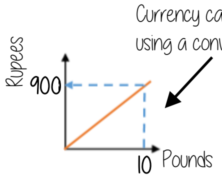
Conversion between currencies (R)

For every £1 I have 90 Rupees

£1 = 90 Rupees ← Currency is directly proportional

x 10 £1 = 90 Rupees x 10
£10 = 900 Rupees

Currency can be converted using a conversion graph



Convert 630 Rupees into Pounds

x 7 £1 = 90 Rupees x 7 ← 630 ÷ 90 = 7
£7 = 630 Rupees

Ratios in 1:n and n:1

This is asking you to cancel down until the part indicated represents 1

Show the ratio 4:20 in the ratio of 1:n

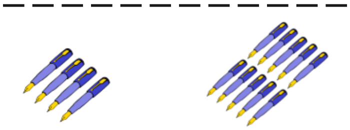
The question states that this part has to be 1 unit. Therefore Divide by 4

4 : 20
1 : 5

This side has to be divided by 4 too – to keep in proportion

the n part does not have to be an integer for this type of question

Best buys



4 pens costs £2.60 | 10 pens costs £6.00

You could work out how much 40 pens are and then compare

"1 pen costs..." £2.60 ÷ 4 = £0.65 | £6.00 ÷ 10 = £0.60

"1-pound buys..." 4 ÷ 2.60 = 1.54 pens | 10 ÷ 6 = 1.67 pens

Compare the solution in the context of the question

The best value has the lowest cost "per pen"

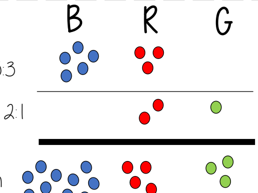
The best value means £1 buys you more pens

Combining ratios

The ratio of Blue counters to Red counters is 5:3

The ratio of Red counters to Green counters is 2:1

Ratio of Blue to Red to Green



10 : 6 : 3

Use equivalent ratios to allow comparison of the group that is common to both statements

Lowest common multiple of the ratio both statements share

YEAR 10 – PROPORTION...

Percentages and Interest

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

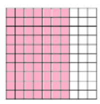
- Convert and compare FDP
- Work out percentages of amounts
- Increase/ decrease by a given percentage
- Express one number as a percentage
- Calculate simple and compound interest
- Calculate repeated percentage change
- Find the original value
- Solve problems with growth and decay

Keywords

Exponent: how many times we use a number in multiplication. It is written as a power
Compound interest: calculating interest on both the amount plus previous interest
Depreciation: a decrease in the value of something over time.
Growth: where a value increases in proportion to its current value such as doubling
Decay: the process of reducing an amount by a consistent percentage rate over time.
Multiplier: the number you are multiplying by
Equivalent: of equal value.

Compare FDP (R)

Comparisons are easier in the same format

$\frac{70}{100}$ → This also means 70 ÷ 100 → 70 out of 100 squares, 70 "hundredths" = 7 "tenths", 0.7 →  → 70 hundredths = 70%

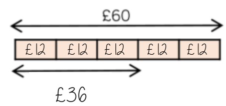
Using a calculator: S⇔D — Convert to a decimal → × 100 converts to a percentage

This will give you the answer in the simplest form

Be careful of recurring decimals
eg $\frac{1}{3}$ = 0.3333333 = $0.\dot{3}$
The dot above the 3

Fraction/ Percentage of amount (R)

Find $\frac{3}{5}$ of £60



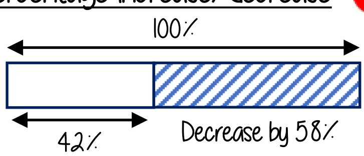
Remember $\frac{3}{5}$ = 60%

10% of £60 = £6
50% of £60 = £30
60% of £60 = £36

Remember $\frac{3}{5}$ = 60% = 0.6

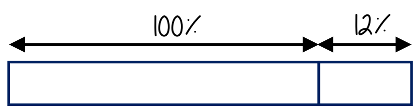
60% of £60
= 0.6 x 60
= £36

Percentage increase/decrease (R)



Decrease by 58%

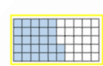
100% - 58% = 42%
100 – 0.58 = 0.42 ← Multiplier Less than 1



Increase by 12%

100% + 12% = 112%
100 + 0.12 = 1.12 ← Multiplier More than 1

Express as a percentage (R)



27 per every 50 shaded: $\frac{27}{50}$ — 54 per every 100 shaded: $\frac{54}{100}$ — 54%

$\frac{13}{30}$ → ■/□ → $\frac{13}{30}$ → × 100 → 43.3333...% → 43%

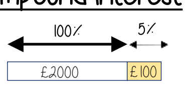
Can't use equivalence easily to find 'per hundred'

Decimal percentages are still a percentage

Simple and compound interest

Simple Interest

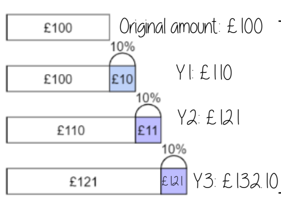
James invests £2000 at 5% simple interest



The original value increases by this amount every year

Compound Interest

Tess invests £100 at 10% compound interest for 3 years



Original amount: £100
Y1: £110
Y2: £121
Y3: £132.10

The multiplier 1.10 repeats each year

Repeated percentage change

Compound Interest

Tess invests £100 at 10% compound interest for 3 years

£100 x 1.10 x 1.10 x 1.10

£100 x 1.10 ^{3}

Original amount — Repeated multiplier — Number of occurrences

Depreciation

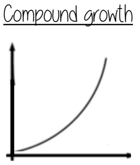
Depreciation calculations use multipliers **less than** 1

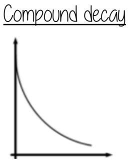
Multipliers are commutative – an overall multiplier effect can be calculated by combining the multipliers separately

eg Increase of 10% then a reduction of 10%: x 1.10 x 0.9 → x 0.99 The multiplier

Growth and decay

Compound growth — Compound decay





Compound growth and compound decay are exponential graphs

Decay – the values get closer to 0
The constant multiplier is less than one

Growth – the values increase exponentially
The constant multiplier is more than one

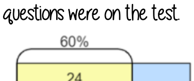
Find the original value

Percentage calculations

Original amount × Multiplier = Final Value

In a test Lucy scored 60% of her questions correctly. Her score was 24. How many questions were on the test

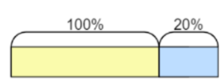
Original x 0.6 = 24



24 ÷ 0.6 = 40 marks
10% = 6
100% = 40

Total questions on test

A car sold for a profit £3000 with a profit of 20%. How much was the car originally?



£3000

Original x 1.2 = 3000

120% = £3000
10% = £250
100% = £2500

YEAR 10 – PROPORTION...

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Probability

What do I need to be able to do?

By the end of this unit you should be able to:

- Add, Subtract and multiply fractions
- Find probabilities using likely outcomes
- Use probability that sums to 1
- Estimate probabilities
- Use Venn diagrams and frequency trees
- Use sample space diagrams
- Calculate probability for independent events
- Use tree diagrams

Keywords

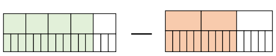
Event: one or more outcomes from an experiment
Outcome: the result of an experiment
Intersection: elements (parts) that are common to both sets
Union: the combination of elements in two sets
Expected Value: the value/ outcome that a prediction would suggest you will get
Universal Set: the set that has all the elements
Systematic: ordering values or outcomes with a strategy and sequence
Product: the answer when two or more values are multiplied together

Add, Subtract and multiply fractions

R

Addition and Subtraction

$$\frac{4}{5} - \frac{2}{3}$$



$$\frac{12}{15} - \frac{10}{15} = \frac{2}{15}$$

Use equivalent fractions to find a common multiple for both denominators

Multiplication

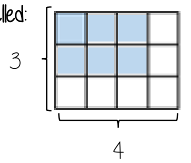
$$\frac{3}{4} \times \frac{2}{3}$$

$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$$

Parts shaded

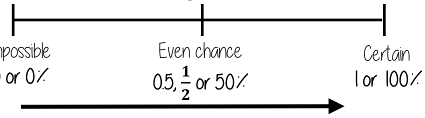
Total number of parts in the diagram

Modelled:



Likeliness of a probability

R



The more likely an event the further up the probability it will be in comparison to another event (It will have a probability closer to 1)

Sum to 1

R

Probability is always a value between 0 and 1

The probability of getting a blue ball is $\frac{1}{5}$

∴The probability of NOT getting a blue ball is $\frac{4}{5}$

The sum of the probabilities is 1

Experimental data

Theoretical probability	What we expect to happen
Experimental probability	What actually happens when we try it out

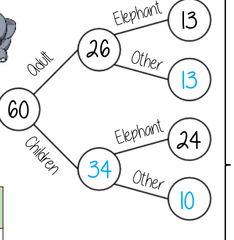
The more trials that are completed the closer experimental probability and theoretical probability become

The probability becomes more accurate with more trials.
Theoretical probability is proportional

Tables, Venn diagrams, Frequency trees

Frequency trees

60 people visited the zoo one Saturday morning. 26 of them were adults. 13 of the adult's favourite animal was an elephant. 24 of the children's favourite animal was an elephant.



Frequency trees and two-way tables can show the same information

The total columns on two-way tables show the possible denominators

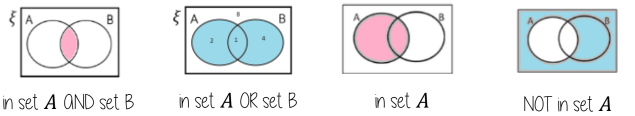
P(adult) = $\frac{26}{60}$

P(Child with favourite animal as elephant) = $\frac{13}{37}$

Two-way table

	Adult	Child	Total
Elephant	13	24	37
Other	13	10	23
Total	26	34	60

Venn diagram



in set A AND set B	in set A OR set B	in set A	NOT in set A
$P(A \cap B)$	$P(A \cup B)$	$P(A)$	$P(A')$

Sample space

The possible outcomes from rolling a dice

R

The possible outcomes from tossing a coin

	1	2	3	4	5	6
H	1,H	2,H	3,H	4,H	5,H	6,H
T	1,T	2,T	3,T	4,T	5,T	6,T

P (Even number and tails) = $\frac{3}{12}$

Independent events

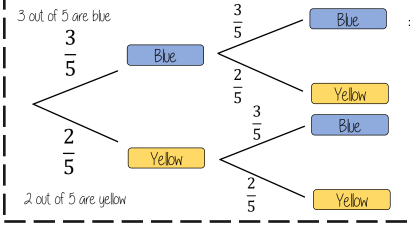
The outcome of two events happening. The outcome of the first event has no bearing on the outcome of the other

$$P(A \text{ and } B) = P(A) \times P(B)$$

Tree diagram for independent event

Isobel has a bag with 3 blue counters and 2 yellow. She picks a counter and replaces it before the second pick.

Because they are replaced the second pick has the same probability



$= P(BB)$ $\frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$

$= P(BY)$ $\frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$

$= P(YB)$ $\frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$

$= P(YY)$ $\frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$

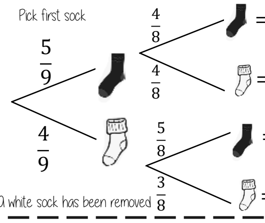
Sum of probabilities = 1

Dependent events

The outcome of the first event has an impact on the second event

Tree diagram for dependent event

A sock drawer has 5 black and 4 white socks, Jamie picks 2 socks from the drawer.



$= P(BB)$ $\frac{5}{9} \times \frac{4}{8} = \frac{20}{72}$

$= P(BW)$ $\frac{5}{9} \times \frac{4}{8} = \frac{20}{72}$

$= P(WB)$ $\frac{4}{9} \times \frac{5}{8} = \frac{20}{72}$

$= P(WW)$ $\frac{4}{9} \times \frac{3}{8} = \frac{12}{72}$

Sum of probabilities = 1

NOTE: as "socks" are removed from the drawer the number of items in that drawer is also reduced ∴ the denominator is also reduced for the second pick

YEAR 10 – DELVING INTO DATA…

Collecting, representing and interpreting

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Construct and interpret frequency tables and polygon. two-way tables, line, bar, & pie charts
- Find and interpret averages from a list and a table
- Construct and interpret time series graphs, stem and leaf diagrams and scatter graphs

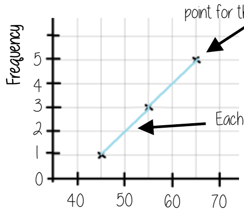
Keywords

Population: the whole group that is being studied
Sample: a selection taken from the population that will let you find out information about the larger group
Representative: a sample group that accurately represents the population
Random sample: a group completely chosen by change. No predictability to who it will include.
Bias: a built-in error that makes all values wrong by a certain amount
Primary data: data collected from an original source for a purpose.
Secondary data: data taken from an external location. Not collected directly.
Outlier: a value that stands apart from the data set

Frequency tables and polygons

x Weight(g)	Frequency
$40 < x \le 50$	1
$50 < x \le 60$	3
$60 < x \le 70$	5

We do not know from grouped data where each value is placed so have to use an estimate for calculations



Each point is plotted at them mid point for the group it represents

Each point is connected with a straight line

The data about weight starts at 40 So the axis can start at 40

MID POINTS
Mid-points are used as estimated values for grouped data. The middle of each group

Mid-point = $\frac{\text{Start point + End point}}{2}$

Two way tables (R)

60 people visited the zoo one Saturday morning. 26 of them were adults. 13 of the adult's favourite animal was an elephant. 24 of the children's favourite animal was an elephant.

Extract information to input to the two-way table.

Subgroups each have their own heading

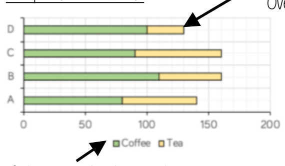
	Adult	Child	Total
Elephant	13	24	37
Other	13	10	23
Total	26	34	60

Needs subgroup totals

Overall total

Bar and line charts

Composite bar charts

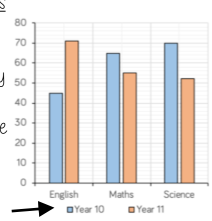


Compare the bars green compared to yellow. The size of each bar is the frequency. Overall total easily comparable

Categories clearly indicated

Dual bar charts

Bars are compared side by side
Easier to compare subgroups



Categories clearly indicated

Draw and interpret Pie Charts (R)

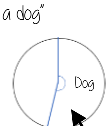
Type of pet	Dog	Cat	Hamster
Frequency	32	25	3

There were 60 people asked in this survey (Total frequency)

$\frac{32}{60}$ "32 out of 60 people had a dog"

This fraction of the 360 degrees represents dogs

$\frac{32}{60} \times 360 = 192°$



Use a protractor to draw This is 192°

Multiple method
As 60 goes into 360 – 6 times.
Each frequency can be multiplied by 6 to find the degrees (proportion of 360)

Comparing Pie Charts: You NEED the overall frequency to make any comparisons

Averages from a table (R)

Non-grouped data

Number of Siblings	0	1	2
Frequency	6	8	6
Subtotal	0	8	12

Overall Frequency: 20

Total number of siblings: 20

The data in a list: 0,0,0,0,0,0,1,1,1,1,1,1,1,1,2,2,2,2,2,2

Mean: $\frac{\text{total number of siblings}}{\text{Total frequency}} = 1$

Grouped data

x Weight(g)	Frequency	Mid Point	MP x Freq
$40 < x \le 50$	1	45	45
$50 < x \le 60$	3	65	195
$60 < x \le 70$	5	65	325

Overall Frequency: 9

Overall Total : 565

Mean: 62.8g

The data in a list: 45, 55, 55, 55, 65, 65, 65, 65, 65

Averages from lists (R)

The Mean

A measure of average to find the central tendency… a typical value that represents the data

24, 8, 4, 11, 8

Find the sum of the data (add the values)
55
Divide the overall total by how many pieces of data you have
55 ÷ 5

Mean = 11

The Mode (The modal value)

This is the number OR the item that occurs the most (it does not have to be numerical)

24, 8, 4, 11, 8

This can still be easier if it the data is ordered first

Mode = 8

The Median

The value in the center (in the middle) of the data

24, 8, 4, 11, 8

Put the data in order 4, 8, 8, 11, 24

Find the value in the middle 4, 8, 8, 11, 24

Median = 8

NOTE: If there is no single middle value find the mean of the two numbers left

For Grouped Data

The modal group – which group has the highest frequency

YEAR 10 – DELVING INTO DATA…

@whisto_maths

Collecting, representing and interpreting

What do I need to be able to do?

By the end of this unit you should be able to:

- Construct and interpret frequency tables and polygon. two-way tables, line, bar, & pie charts
- Find and interpret averages from a list and a table
- Construct and interpret time series graphs, stem and leaf diagrams and scatter graphs

Keywords

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Stem and leaf

A way to represent data and use to find averages

This stem and leaf diagram shows the age of people in a line at the supermarket.

0	7 9
1	4 5 6 8 8
2	1 3
3	0

Key: 1|4 Means 14 years old

Stem and leaf diagrams:
Must include a key to explain what it represents
The information in the diagram should be ordered

Back to back stem and leaf diagrams

Girls		Boys
5	14	
7, 5, 5, 5, 4	15	3, 8, 9
8, 4, 2, 1, 0	16	2, 5, 7, 7, 7, 8, 8, 9
9, 8, 7, 6, 6, 4, 2, 1, 1, 0, 0	17	0, 2, 3, 6, 6, 7, 7
	18	0, 1, 4, 5

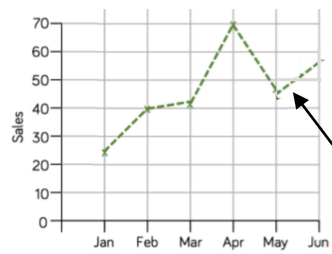
15 | 3,
Means 153 cm tall

Back to back stem and leaf diagrams
Allow comparisons of similar groups
Allow representations of two sets of data

Time-Series

R

This time-series graph shows the total number of car sales in £1000 over time



Look for general trends in the data. Some data shows a clear increase or a clear decrease over time.

Readings in-between points are estimates (on the dotted lines). You can use them to make assumptions.

Comparing distributions

Comparisons should include a statement of average and central tendency, as well as a statement about spread and consistency.

Mean, mode, median – allows for a comparison about more or less average
Range – allows for a comparison about reliability and consistency of data

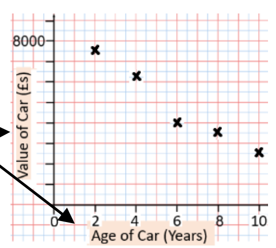
Draw and interpret a scatter graph

R

Age of Car (Years)	2	4	6	8	10
Value of Car (£s)	7500	6250	4000	3500	2500

- This data may not be given in size order
- The data forms information pairs for the scatter graph
- Not all data has a relationship

All axes should be labelled



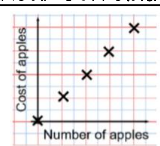
The axis should fit all the values on and be equally spread out

"This scatter graph show as the age of a car increases the value decreases"

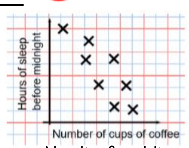
The link between the data can be explained verbally

Linear Correlation

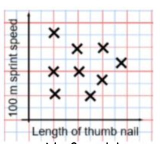
R



Positive Correlation



Negative Correlation



No Correlation

As one variable increases so does the other variable

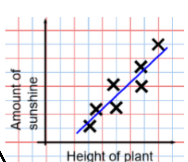
As one variable increases the other variable decreases

There is no relationship between the two variables

The line of best fit

R

The Line of best fit is used to make estimates about the information in your scatter graph



Things to know:

- The line of best fit DOES NOT need to go through the origin (The point the axes cross)
- There should be approximately the same number of points above and below the line (It may not go through any points)
- The line extends across the whole graph

It is only an estimate because the line is designed to be an average representation of the data

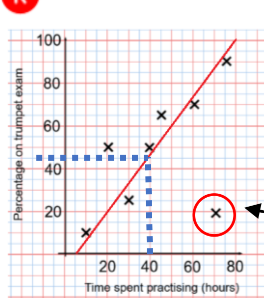
It is always a straight line.

Using a line of best fit

R

Interpolation is using the line of best fit to estimate values inside our data point.

e.g. 40 hours revising predicts a percentage of 45



Extrapolation is where we use our line of best fit to predict information outside of our data
This is not always useful – in this example you cannot score more that 100%. So revising for longer can not be estimated

This point is an "outlier"
It is an outlier because it doesn't fit this model and stands apart from the data

YEAR 10 – USING NUMBER...

Non-calculator methods

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What do I need to be able to do?

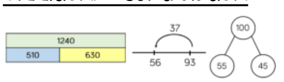
By the end of this unit you should be able to:

- Use mental/written methods for the four number operations
- Use four operations for fractions
- Write exact answers
- Round to decimal places and significant figures
- Estimate solutions
- Understand limits of accuracy
- Understand financial maths

Keywords

Truncate: to shorten, to shorten a number (no rounding), to shorten a shape (remove a part of the shape)
Round: making a number simpler, but keeping its place value close the what it originally was
Credit: money that goes into a bank account
Debit: money that leaves a bank account
Profit: the amount of money after income - costs
Tax: money that the government collects based on income, sales and other activities.
Balance: The amount of money in a bank account
Overestimate: Rounding up – gives a solution higher than the actual value
Underestimate: Rounding down – gives a solution lower than the actual value

Addition/ Subtraction



Modelling methods for addition/ subtraction

- Bar models
- Number lines
- Part/ Whole diagrams

Addition is commutative

6 + 3 = 3 + 6

The order of addition does not change the result

Subtraction the order has to stay the same

360 – 147 = 360 – 100 – 40 – 7

- Number lines help for addition and subtraction
- Working in 10's first aids mental addition/ subtraction
- Show your relationships by writing fact families

Formal written methods

	H	T	O
	1	8	7
+	5	4	2

	H	T	O
	4	2	7
–	2	4	9

Remember the place value of each column. You may need to move 10 ones to the ones column to be able to subtract

Decimals have the same methods remember to align the place value

Division methods

Short division

$3584 \div 7 = 512$

```
    5  1  2
7 | 3 ³5  8 ¹4
```

Complex division

$\div 24 = \div 6 \div 4$

Break up the divisor using factors

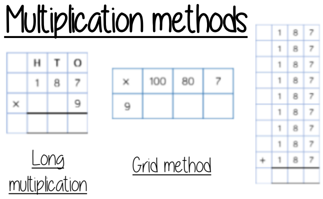
Division with decimals

The placeholder in division methods is essential – the decimal lines up on the dividend and the quotient

$24 \div 0.02 \longrightarrow 24 \div 0.2 \longrightarrow 240 \div 2$

All give the same solution as represent the same proportion.
Multiply the values in proportion until the divisor becomes an integer

Multiplication methods



Long multiplication (column)

Grid method

Repeated addition

Less effective method especially for bigger multiplication

Multiplication with decimals

Perform multiplications as integers
eg $0.2 \times 0.3 \longrightarrow 2 \times 3$

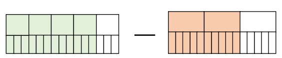
Make **adjustments** to your answer to match the question: $0.2 \times 10 = 2$
$0.3 \times 10 = 3$

Therefore $6 \div 100 = \underline{0.06}$

Four operations with fractions

Addition and Subtraction

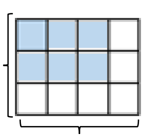
$\frac{4}{5} - \frac{2}{3}$



$\frac{12}{15} - \frac{10}{15} = \frac{2}{15}$

Multiplication

$\frac{3}{4} \times \frac{2}{3}$



$= \frac{6}{12} = \frac{1}{2}$

Division

$\frac{2}{5} \div \frac{3}{4}$

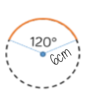
$\frac{2}{5} \times \frac{4}{3}$

Multiplying by a reciprocal gives the same outcome

$= \frac{8}{15}$

Exact Values

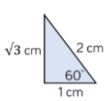
Leave in terms of π



$= \frac{120}{360} \times 36\pi$

$= \frac{1}{3} \times 36\pi = \underline{12\pi}$

Leave as a surd



$\text{Tan}30 = \frac{1}{\sqrt{3}}$

Rounding

2.46192 (to 2dp) - Is this closer to 2.46 or 2.47

2.46 2.47

2.46 | 192

This shows the number is closer to 2.46

Significant Figures

370 to 1 significant figure is 400
37 to 1 significant figure is 40
3.7 to 1 significant figure is 4
0.37 to 1 significant figure is 0.4
0.00000037 to 1 significant figure is 0.0000004

SF: Round to the first nonzero number

Estimation

Round to 1 significant figure to estimate

$21.4 \times 3.1 \approx 20 \times 3 \approx 60$

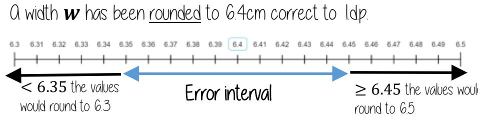
The equal sign changes to show it is an estimation

This is an **underestimate** because both values were rounded down

It is good to check all calculations with an estimate in all aspects of maths – it helps you identify calculation errors.

Limits of accuracy

A width w has been rounded to 6.4cm correct to 1dp.

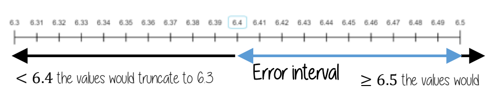


The error interval

$6.35 \leq w < 6.45$

Any value within these limits would round to 6.4 to 1dp

A width w has been truncated to 6.4cm correct to 1dp.



$6.4 \leq w < 6.5$

Any value within these limits would truncate to 6.4 to 1dp

YEAR 10 – USING NUMBER...

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Types of number & sequences

What do I need to be able to do?

By the end of this unit you should be able to:

- Understand factors and multiples
- Express numbers as a product of primes
- Find the HCF and LCM
- Describe and continue sequences
- Explore sequences
- Find the nth term of a linear sequence

Keywords

Factor: numbers we multiply together to make another number
Multiple: the result of multiplying a number by an integer.
HCF: highest common factor. The biggest factor that numbers share.
LCM: lowest common multiple. The first multiple numbers share.
Arithmetic: a sequence where the difference between the terms is constant
Geometric: a sequence where each term is found by multiplying the previous one by a fixed nonzero number
Sequence: items or numbers put in a pre-decided order

Multiples

The "times table" of a given number

All the numbers in this lists below are multiples of 3.

3, 6, 9, 12, 15...

This list continues and doesn't end

$3x, 6x, 9x$ ...

x could take any value and as the variable is a multiple of 3 the answer will also be a multiple of 3

Non example of a multiple

4.5 is not a multiple of 3 because it is 3 x 1.5 ← Not an integer

Factors

Arrays can help represent factors

5 x 2 or 2 x 5

Factors of 10: 1, 2, 5, 10

10 x 1 or 1 x 10

The number itself is always a factor

Factors and expressions

$6x \times 1$ OR $6 \times x$

Factors of $6x$: $6, x, 1, 6x, 2x, 3, 3x, 2$

$2x \times 3$

$3x \times 2$

Prime numbers

- Integer
- Only has 2 factors
- and itself

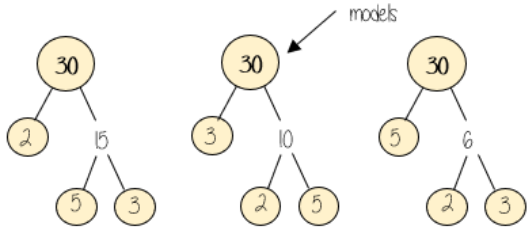
2 — The first prime number. The only even prime number

Learn or how-to quick recall...

2, 3, 5, 7, 11, 13, 17, 19, 23, 29...

Product of prime factors

Multiplication part-whole models



All three prime factor trees represent the same decomposition

30 = 2 x 3 x 5 ← Multiplication of prime factors

Using prime factors for predictions

eg **60**	30 x 2	2 x 3 x 5 **x 2**
150	30 x 5	2 x 3 x 5 **x 5**

Finding the HCF and LCM

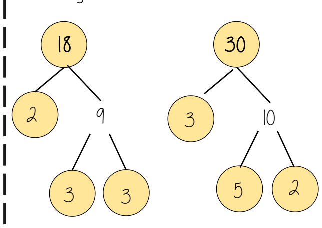
HCF – Highest common factor

HCF of 18 and 30

18: 1, 2, 3, 6, 9, 18

30: 1, 2, 3, 5, 6, 10, 15, 30

6 is the biggest factor they share — HCF = 6



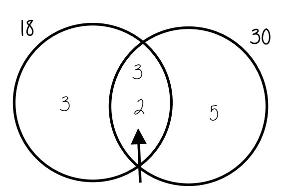
LCM – Lowest common multiple

LCM of 18 and 30

18: 18, 36, 54, 72, 90

30: 30, 60, 90

The first time their multiples match — LCM = 90



Arithmetic/ Geometric sequences

Arithmetic Sequences change by a common difference. This is found by addition or subtraction between terms

Geometric Sequences change by a common ratio. This is found my multiplication/ division between terms.

Term to term rule – how you get from one term (number in the sequence) to the next term.

Position to term rule – take the rule and substitute in a position to find a term. Eg. Multiply the position number by 3 and then add 2

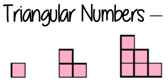
Other sequences

Fibonacci Sequence

1, 1, 2, 3, 5, 8 ...

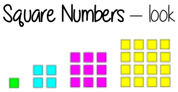
Each term is the sum of the previous two terms

Triangular Numbers – look at the formation



1, 3, 6, 10, 15 ...

Square Numbers – look at the formation



1, 4, 9, 16...

Sequences are the repetition of a patten

Finding the nth term

This is the 4 times table → 4, 8, 12, 16, 20...

4n

7, 11, 15, 19, 22

This has the same constant difference – but is 3 more than the original sequence

4n + 3

This is the constant difference between the terms in the sequence

This is the comparison (difference) between the original and new sequence

YEAR 10 – USING NUMBER...

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Indices & Roots

What do I need to be able to do?

By the end of this unit you should be able to:

- Identify square and cube numbers
- Calculate higher powers and roots
- Understand powers of 10 and standard form
- Know the addition and subtraction rule for indices
- Understand power zero and negative indices
- Calculate with numbers in standard form

Keywords

Standard (index) Form: A system of writing very big or very small numbers

Commutative: an operation is commutative if changing the order does not change the result.

Base: The number that gets multiplied by a power

Power: The exponent – or the number that tells you how many times to use the number in multiplication

Exponent: The power – or the number that tells you how many times to use the number in multiplication

Indices: The power or the exponent.

Negative: A value below zero.

Coefficient: The number used to multiply a variable

Square and cube numbers

Square numbers

1, 4, 9, 16...

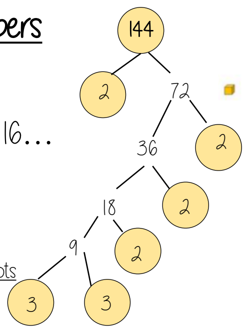
144 = 2 x 2 x 2 x 2 x 3 x 3

2 x 2 x 3 x 2 x 2 x 3

12 x 12

Prime factors can find square roots

$\sqrt{144} = 12$



Cube numbers

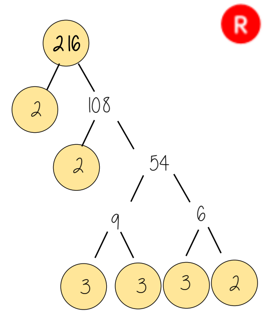
1, 8, 27, 64, 125...

216 = 2 x 2 x 2 x 3 x 3 x 3

2 x 3 x 2 x 3 x 2 x 3

6 x 6 x 6

$\sqrt[3]{216} = 6$



Higher powers and roots

x^n — n – power (number of times multiplied by itself)

x – the base number.

$\sqrt[n]{x}$ — Finding the nth root of any value

Other mental strategies for square roots

$\sqrt{810000} = \sqrt{81} \times \sqrt{10000}$

$= 9 \times 100$

$= 900$

Standard form

$A \times 10^n$

Any number between 1 and less than 10

Any integer

0.001

$1 \times \frac{1}{1000}$

1×10^{-3}

10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^1	10^0	10^{-1}	10^{-2}	10^{-3}
10	1	0.1	0.01	0.001

Any value to the power 0 always = 1

Negative powers do not indicate negative solutions

Numbers in standard form with negative powers will be less than 1

Example

3.2×10^4

$= 3.2 \times 10 \times 10 \times 10 \times 10$

$= 32000$

Non-example

0.8×10^4

$5.3 \times 10^{0.7}$

$3.2 \times 10^{-4} = 3.2 \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = 0.00032$

Addition/ Subtraction Laws

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m-n}$

Zero and negative indices

$x^0 = 1$

Any number divided by itself = 1

$\frac{a^6}{a^6} = a^6 \div a^6$

$= a^{6-6} = a^0 = 1$

Negative indices do not indicate negative solutions

$2^2 = 4$

$2^1 = 2$

$2^0 = 1$

$2^{-1} = \frac{1}{2}$

$2^{-2} = \frac{1}{4}$

Looking at the sequence can help to understand negative powers

Powers of powers

$(x^a)^b = x^{ab}$

$(2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3$

The same base and power is repeated. Use the addition law for indices

$(2^3)^4 = 2^{12}$ ← $a \times b = 3 \times 4 = 12$

NOTICE the difference

$(2x^3)^4 = 2x^3 \times 2x^3 \times 2x^3 \times 2x^3$

The addition law applies ONLY to the powers. The integers still need to be multiplied

$(2x^3)^4 = 16x^{12}$

Standard form calculations

Addition and Subtraction

Tip: Convert into ordinary numbers first and back to standard from at the end

$6 \times 10^5 + 8 \times 10^5$

Method 1

$= 600000 + 800000$

$= 1400000$

$= 1.4 \times 10^6$

Method 2

$= (6 + 8) \times 10^5$

$= 14 \times 10^5$

$= 1.4 \times 10^1 \times 10^5$

$= 1.4 \times 10^6$

This is not the final answer

Multiplication and division

$\frac{1.5 \times 10^5}{0.3 \times 10^3}$

Division questions can look like this

$(1.5 \times 10^5) \div (0.3 \times 10^3)$

$1.5 \div 0.3 \times 10^5 \div 10^3$

$= 5 \times 10^2$

For multiplication and division you can look at the values for A and the powers of 10 as two separate calculations